

TEACHERS' REVIEW OF TASKS AS A TOOL FOR EXAMINING SECONDARY TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING

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One important aspect of teaching is reviewing tasks in preparation for instruction. The goal of the multicase study of four secondary teachers was to examine the interplay between their mathematical knowledge for teaching [MKT] and what they attend to when reviewing a mathematics task. We engaged secondary mathematics teachers in a semi-structured, clinical interview focused on a non-routine mathematical task involving exponential growth. The results suggest experienced teachers may not explicitly attend to learning opportunities in their review of a task, and their own mathematical work contributes to their anticipation of student work and thinking. This work highlights how researchers focused on MKT can use clinical interviews as a tool for extracting and describing a teacher's MKT.

Keywords: Mathematical Knowledge for Teaching; Instructional Activities and Practices; Curriculum Enactment

Mathematical knowledge for teaching [MKT] is the “knowledge needed to carry out the work of teaching mathematics” (Ball, Thames, & Phelps, 2008, p. 395). Ball et al. (2008) clarify that *teaching mathematics* includes “everything that teachers must do to support the learning of their students” (p. 395). As such, this work includes planning, instruction, and assessment. Others in the literature suggest, in agreement, that pedagogy, the curriculum, and teachers’ mathematical understanding are interconnected (e.g., Davis & Simmt, 2006; Sullivan, Knott, & Yang, 2015). For example, Sullivan et al. (2015) argue that “tasks do not exist separately from the pedagogies associated with their use nor are the pedagogies independent of the task” (p. 84).

One aspect of the mathematical work of teaching is reviewing mathematical tasks for potential use during instruction (Ball, 2017). From selection to implementation, teachers’ use of mathematical tasks impacts the types of learning opportunities their students experience (e.g., Stein, Grover, & Henningsen, 1996). Sullivan et al. (2015) note that tasks allow students the opportunity to experience mathematical concepts and ideas. They claim that “the role of the teacher is to select, modify, design, redesign, sequence, implement, and evaluate tasks” (p. 83). Furthermore, “in planning and teaching, the role of the teacher is to identify potential and perceived blockages, prompts, supports, challenges, and pathways” (Sullivan et al., 2015, p. 86).







The purpose of this research study was to explore the aspects of a teacher’s MKT elicited when reviewing a mathematics task. Specifically, we claim that aspects of a teacher’s MKT including specialized content knowledge [SCK], knowledge of content and teaching [KCT], and knowledge of content and students [KCS] (Ball et al., 2008) become evident during task analysis. To investigate our claim, we pursued the following research question: What aspects of a secondary mathematics teachers’ MKT can we describe from their review of a nonstandard exponential functions task? Our focus on exponential functions answers a call for increased research considering topics at the secondary level (Speer, King, and Howell, 2015).

Methods

The study was part of a larger research study focused on the work of 16 high school teachers from the Great Plains region of the United States while they were teaching exponential functions in courses ranging from Algebra I to Precalculus. The research team identified the teachers as “highly effective” based on recommendations from the teachers’ administrators or peers. The teachers had 5 to 25 years of teaching experience.

Data collection for the larger study consisted of multiple stages. First, a member of the research team conducted classroom observations of five lessons pertaining to exponential functions. In addition to observations, the teachers engaged in pre- and post-lesson interviews focused on the entire set of lessons as well as pre- and post-lesson interviews for each lesson. Second, a member of the research team administered a pre- and post-lesson assessment measuring the teachers’ students’ understanding on exponential functions topics. Finally, each teacher engaged in a semi-structured, clinical interview, called the *MKT Interview*, which focused on the teachers reviewing two non-standard exponential functions tasks. In this paper we focus on the first task of the MKT Interview, the *Xbox Xponential* task (see Figure 1).

Part 1. In 1965, computer scientist Gordon Moore predicted that computer processor speeds would double every two years. Twelve years later, Atari released the 2600 with a processor speed of 1.2 MHz. Based on Moore’s law, how fast would you expect the processors to be in each of the consoles below?

 Atari 2600 1977 1.2 MHz	 Intellivision 1979	 N.E.S. 1983	 Atari Jaguar 1993	 GameCube 2001	 XBOX 360 2005
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Part 2. Assuming Moore’s Law is true, write an expression to estimate how fast console processors should be in 2077, a century after the original Atari.

Part 3. We can think of the release of the Atari 2600 as the start of the video game era: 1977 CE becomes year 0 VG. Let the variable t represent the number of years that have passed since 1997, i.e., the video game year. Write an equation to calculate the expected processor speed for a given video game year.

Figure 1: The Xbox Xponential task (modified from Mathelicious, 2015).

During a semi-structured, clinical interview, a member of the research team asked the teacher to review the *Xbox Xponential* task. The teachers were prompted to articulate the mathematical opportunities to learn the task had the potential to support if used during classroom instruction. During the interview, the researchers collected video data of the teacher engaging with the task. Data analysis focused on how teachers approached the task mathematically and the descriptions they used to express the ways they thought students would interact with the task. We chose the Xbox task because it supported students to think about key mathematical ideas related to exponential functions and contained nonstandard elements. Specifically, the task provided opportunities for students to think about how a change in the independent variable other than one unit impacts the change in the dependent variable and how to capture that change in a table, an expression, and an equation. We anticipated that learners would work through calculating specific values using a multiplicative relationship between dependent and independent variables using recursive reasoning (part one), generalizing their work into an expression for a specific year (part two), and creating a general equation for the relationship (part three). This work provided the opportunity for students to learn about: the connections between multiplicative growth and exponential functions, the connections between repeated multiplication and exponents, and the importance of defining independent and dependent variables.

The Cases

Guided by an interpretive theoretical perspective (Creswell, 2013), we selected four teachers who engaged with the Xbox Xponential task during the clinical MKT interview in this multicase study (Stake, 2006). As part of our theoretical perspective, our participants developed “subjective meanings of their experiences,” and our role as researchers was to interpret our participants’ responses by relying “as much as possible on the participants’ views” (Creswell, 2013, pp. 24-25). However, our analysis included interpretations and observations to investigate the teachers’ MKT. The individual cases of Helen, Abby, Frankie, and Molly provided contrasting views of the Xbox Task and their cross-case analysis provided evidence of their collective MKT.

Helen

For part one of the task, Helen immediately identified the gaps in time and indicated them with arrows and increments from cell to cell in the table. For example, from 1983 to 1993, she drew an arrow and wrote “+10” to indicate the 10-year gap in time. She noted that all of the gaps in time are even which made “them all nice and easy” because “you [do] not have to deal with what happens if it is not an even number.” After determining the gaps in time, Helen used a recursive strategy to find the values for the processing speeds by multiplying the previous value by a power of two based on the number of two-year intervals. For part two of the task, Helen initially wrote $y = 1.2 \cdot 2^{100}$. After moving on to part three of the task, she returned to part two to modify her answer to $f(100) = 1.2 \cdot 2^{100/2}$ which she simplified to $f(100) = 1.2 \cdot 2^{50}$. Finally, for part three of the task, she wrote her answer as $f(t) = 1.2 \cdot 2^{t/2}$. Helen completed all parts of the task as designed before addressing what learning opportunities were possible. With the exception of Helen’s comment about the gaps in time in the table for part one being “nice and easy,” Helen did not comment on her thinking while completing the task. It is interesting to note that Helen was able to attend to the two-year gaps and intervals in part one of the task, but she did not initially attend to them in part two of the task. It was only after moving on to part three that she returned to correct her answer.

With respect to the sequence of tasks, Helen claimed that “it’s always easier for students to handle the numeric at first, especially if it’s their first introduction to exponential functions.” Starting with the table supports this view. She believed that her students would be able to “reason their way through” the table by attending to the two-year intervals and gaps in time. While some students might use a “brute force” strategy of repeatedly multiplying by two, Helen hoped that the table would motivate her students “to start thinking of another way.” By doing that, she claimed that this “would allow them to make that jump into actually formalizing [the context] and writing it algebraically in general.” Helen sees exponents as a tool students could use to complete the task. Helen noted that the sequence of tasks supports students at all levels to be able to complete the task since “not all students would be able to start [with part three].” The sequence gives all students “a path towards getting the ultimate goal of the [task] which would be coming up with [the function in part three].” Throughout these comments Helen is primarily focusing on students’ completion of the task and the prior knowledge they need for successful completion of the task.

Helen highlighted that repeated multiplication is “a huge aspect of why we use exponential functions.” She viewed the table as potentially supporting her in teaching students “about the behavior of exponential functions.” Specifically, “the idea that you are doubling every certain number of years.” In contrast to her previous comments, Helen, is now focused on student thinking about a key mathematical idea. Her comments stemmed from her view that the doubling is occurring every two years is “an interesting twist from the [tasks] that [her students] may be used to seeing.” She hypothesized that her students would not recognize that they would need to divide the value in the exponent by two which is the same mistake she initially made when completing the task herself.

Abby

For part one of the task, Abby filled in the table by repeatedly multiplying the value in the previous table cell by two, ignoring the gaps in time. After completing the table in this way, she moved on to part two. She first wrote the equation, $f(x) = 1.2(2)^{x-1}$. Then, when she attempted to define her variable, x , she realized her error in part one. She then returned to part one of the task by identifying the size of the gaps in the years: +2, +4, +10, +8, and +4 respectively. Then, for each of the table cells, she identified the number of two-year intervals and multiplied the previous values by two raised to the exponent related to that number of intervals. For example, from 1983 to 1993 she multiplied 4.8 by 2^5 in order to calculate the value associated with 1993, or ten years after 1983. Later in the interview, Abby admitted that she did not attend to the years and assumed that they were “nice equal” intervals.

After completing the table for the second time, Abby returned to her earlier work where $f(x) = 1.2(2)^{x-1}$ was already written on the paper. Abby then defined x as “every two years.” After asking herself, “does that work?” and checking values, Abby erased the original exponent $x - 1$ as well as her definition for the variable x . Looking back at the table, Abby changed the definition of the variable x to “# of yrs since 1997” and changed the exponent to $x/2$. Again, Abby checked her work by evaluating using her equation and comparing the values with the table in part one of the task. After reading the prompt for part three of the task, Abby noted that she “should read the question” for part two of the task. For her answer, Abby wrote and calculated $f(100) = (1.2)(2^{50}) = 1.251 \times 10^{15}$ for part two of the task. She then moved on to part three of the task where she wrote $f(t) = 1.2(2)^{t/2}$ and defined the variable t as “# of yrs since 1997.” Abby appeared to skim the task initially and completed what she assumed the parts of the task were asking. It was only after moving on to subsequent parts did Abby identify that she may have made an error. This suggests that Abby assumed that the task followed the format of (1) fill in the table for consecutive values, then (2) write an equation to model the context.

Abby noted that the overall sequence of tasks allowed students to access the mathematics at different levels. First, she liked “that we can model it numerically with the table.” This is something that the students “could get just with a little calculator work and it is something that [the students] could actually conceptualize.” Second, she noted that the task is an easy context to understand which would motivate students to complete it. Finally, the sequence of tasks “stair steps” students through the parts towards an answer and “gives them a method to check their work as they go.” Specifically, the table allows students to see that they should divide by two in part two of the task as well as provides them with a way to check their function in part three of the task. Overall, Abby viewed the goal of the task as formulating a model which she claimed is done in part three of the task. She did not articulate any mathematical ideas that students would have the opportunity to think about through engagement with the task.

Abby noted that she could use the task to highlight the connections between the context and the actual real-world data. She noted that the task “would definitely teach [the students] to pay attention to their data.” This mirrors the mistake Abby made while completing the task. Using this task, she might ask students to determine whether Moore’s Law is true. Abby emphasized the use of graphing and using the table to check answers as strategies she would use to teach the task as well as a strategy for students to complete the task; again, highlighting Abby’s focus on what students will do.

Frankie

Frankie only completed portions of the task for herself, and only after being prompted to do so by the interviewer. Frankie never filled in values for the table in part one of the task. However, she noted that, in order to complete the table, a rule like 1.2×2^x would be helpful and “is almost demanded” by the task design. In part two of the task, Frankie used her previously identified rule as a

guide to find the answer $S_{2077} = 1.2 \times 2^{100/2}$. Frankie viewed part three of the task as a more formal version of the rule she constructed earlier. Specifically, the answer to part three of the task was $S(t) = 1.2 \times 2^t$ with the caveat that the t needed to be “adjusted.” That is, “if t is the number of years, then you also have to have the idea that it’s going to be divided by two.” As a result, she rewrote her answer as $S(t) = 1.2 \times 2^{t/2}$.

Frankie highlighted the gaps in time for the various gaming systems. She noted that her students would likely need experience with problems that contain varying gaps in time to be successful in completing the task. In particular, she believed that her students’ “tendency would be to just double and not pay attention to the years and how far apart they [are].” For those students who did attend to the gaps in years, Frankie expressed concern that they would get “hung up” on the 10-year jump. She believed that the table “is not necessarily going to get [the students] to an expression.” To complete the table, the students might need “some intermediate values” in the table in addition to “really think[ing] about how the doubling is happening and how many doublings would take place.” Frankie highlighted that students do not encounter tasks with interval gaps in other areas of the curriculum with the exception of linear functions. As a result, it would not be something that the students would immediately notice.

Frankie did not see a connection between part one of the task and part two of the task which might stem from the fact that she never completed the task as designed. As noted before, Frankie used a rule to support writing the answer to part two of the task. She claimed that her students would need to have “recogniz[ed] that doubling piece” in order to find the answer to part two of the task. While Frankie acknowledged that her students would need to have identified that the doubling was occurring over a two-year span, she did not connect part one of the task as supporting this realization in her students. If students successfully identified the doubling was occurring every two years, she believed some of her students would simplify their answers to part two of the task to “be two to the 50th power.” Other students, she believed would simply raise to the 100th power without dividing by two. In her experience, her students, “when given something like 1.2×2^x , [the students] know to put a number in there and to get something.” She noted that her students do not consider how “the power is changing.” Throughout her comments Frankie is focused on the aspects of the task students are familiar or unfamiliar with and how they will respond.

Frankie viewed the purpose of the task as writing an example of a known formula type which matches the given data. She argued that “adjust[ing]” the power in part three of the task was the trickiest part of the overall task. She believed that her students would be able to work through parts one and two of the task “pretty well” and would not necessarily need to recognize “that any number of years is going to have to have [an] adjustment.” While Frankie thought that students at “several levels” could be successful, she was concerned about students getting “caught up in the information” which would “keep them from moving forward.”

Molly

Prior to starting the task Molly said that she had seen the task before when looking for material. When Molly began working on part one, she noted the varying gaps in years given in the table and said “this would give students an opportunity to start thinking about doubling periods.” Molly suggested that she might have structured the table to start at year 0 and use the number of years since 1977 instead of using the actual years. As she continued to discuss student thinking around the table Molly created a table with “# of doubling periods” as the independent variable which she completed for three doubling periods. She transferred her work from the created table to the table provided in the task. Even though Molly said that she would present the information differently, she liked the way the task designers chose to present the information. Molly thought that the structure of the table would cause students to think about doubling periods as opposed to years. She said, “I think they’re

just thinking about doubling periods and they're working with exponents without realizing that's what they're working with." Molly did not complete the remainder of part one and it did not appear that Molly completed any of part the task for herself; rather, she the work she did was in service of illustrating the thinking she expected students to engage in.

When she began part two, Molly underlined "write an expression" and "a century." While discussing why the task designers included "a century," Molly initially seemed to misinterpret the task, she said "a century, ok and then they've given them another little hint here, so they're telling them that even though 2077 isn't, oh no, I'm sorry I was going back to 1965, so maybe that's what a student would do too." It seemed as if Molly had not thoroughly read the task and assumed that the initial processor speed was given for the year 1965. After clarifying the instructions Molly wrote the expressions $1.2(2)^{100/2}$ and $1.2(2)^{50}$. As with part one, Molly's work appeared to be in service of illustrating potential student thinking. She said the thinking in part two was similar to the thinking required for part one because students needed to focus on the number of doubling periods. She expected students to write $1.2(2)^{50}$ instead of $1.2(2)^{100/2}$ because "they're not going to want to see a fraction there so they would think about, it's doubled, I had to multiply it by two, fifty times." Molly said students might struggle with seeing how to come up with a pattern based on the starting point because they have to move from doing the problem recursively in part one to needing to base their expression off of a starting point in part two.

For part three, Molly underlined "write an equation for the expected processor speed for a given year" and then wrote $y = 1.2(2)^{(t-1977)/2}$ while explaining that part two can be generalized by starting with an original value and "multiplying by two a whole bunch of times." Molly explained the exponents as subtracting 1977 from the year you are looking for and then dividing by two. The equation mimics Molly's process in calculating the processor speeds. Only after the interviewer asked Molly a question which prompted her to reread the task did she write $y = 1.2(2)^{t/2}$ as her final answer for part three. Again, indicating that Molly did not thoroughly read the task but worked off of assumptions about the nature of the task. Molly summarized the overall structure of the task as, "from scaffolding from something that's very simple, just coming up with number answers ... I need fifty years in the future, so I'm not going to be able to just keep extending my chart to get that ... and so now I needed to take this and extend it to something where I can generalize." Molly said that this sort of task structure was common.

Cross Cases Analysis and Discussion

With this research study, we sought to describe aspects of a teacher's MKT based on what they attend to when reviewing a mathematics task for potential use during instruction. We claim that the previous case summaries provide rich opportunities to explore and describe aspects of the teachers' MKT including specialized content knowledge [SCK], knowledge of content and teaching [KCT], and knowledge of content and students [KCS] (Ball et al., 2008).

Ball et al. (2008) identified SCK as the "mathematical knowledge not typically needed for purposes other than teaching" (p. 400). This knowledge includes the ability to identify and interpret student mathematical work as well as the ability to make "features of particular content visible to and learnable by students" (Ball et al., 2008, p. 400). We chose this task partially because the relationship "doubling every two years" has the potential to make important mathematics visible to students. Specifically, doubling indicates that the relationship is exponential while the "every two-year period" adds a unique difficulty when compared to tasks the teachers would typically use. In the previous summaries, we saw that all of the teachers noted that "doubling every two years" was a significant part of the task. The ways in which the teachers spoke about the significance of "doubling every two years" revealed interesting aspects of their SCK.

Helen saw the gaps in the years as something students needed to attend to in order to complete the table. She anticipated that students may not realize the number of years passed needed to be divided by two. She hoped the gaps would encourage students to use exponents as a push towards writing a function. Here she seemed to be implying something about student thinking but this was not explicit. When Helen talked about doubling, she explicitly discussed student thinking saying that students will understand something about the nature of exponential functions. Here we see that Helen's SCK allowed her to see the usefulness of the two-year gaps as supporting her students towards using exponents and writing an equation.

In contrast, Molly discussed the two-year gaps as doubling periods. Unlike the other teachers Molly did not anticipate the two-year gaps to be an issue for students. Rather she described students as finding the number of doubling periods for the two- and four-year gaps without necessarily realizing that they are dividing the number of years by two. Molly believed that students would realize that two years is one doubling period and that four years is two doubling periods. Molly thought that students might have more difficulty in thinking about the number of doubling periods for the ten-year gap. From Molly's interview, it is clear she saw the structure of the table as a key aspect of the task because it supports students in thinking about doubling periods. Molly's identification of aspects of the task which makes visible the key mathematical idea of doubling periods of exponential functions is an important component of SCK.

KCT "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008, p. 401). For example, Ball et al. (2008) noted that teachers must "choose which examples to start with and which examples to use to take students deeper into the content" (Ball et al., 2008, p. 401). With respect to the task design, Abby, Molly, and Helen commented that they liked how the sequencing of the task provided scaffolding towards the equation and that starting with numerical calculations was easier for students. They all saw the goal of the task as doing something (writing a function) rather than thinking about some key mathematical ideas. This suggests that the three teachers KCT includes the idea that students can develop models of exponential growth from exploring values in a table, then calculating a larger value, and finally writing a formalized equation.

In contrast to Abby, Helen, and Molly, Frankie did not see the first part of the task as supporting students in completing the later parts of the task. For example, she identified the ten-year gap as especially significant as an obstacle that could prevent successful completion of the task. Frankie claimed that having a rule is almost required to complete the table and frequently discussed prior experience students would need to be successful in completing the task. Frankie's view of the purpose of the task was slightly different from Abby, Helen, and Molly. Where Abby, Helen and Molly saw the purpose as writing an equation based on the work done to create the table, Frankie saw the purpose as fitting a known equation type to the situation. This suggests that Frankie's KCT, as elicited by the task and the interview, does not include the same construct that the other teachers' have. Frankie's view of the purpose may be a consequence of her choice to write an equation to solve the first part of the task instead of solving the task as written. Her reliance on her equation to complete the table may have prevented her from seeing the ways that the table supported students to write an expression in part 2 and an equation in part 3.

Ball et al. (2008) defined KCS as the "knowledge that combines knowing about students and knowing about mathematics" (p. 401). This knowledge includes an understanding of the content so that the teacher can identify what has the potential to be confusing, challenging, easy, motivating, and interesting for students (Ball et al., 2018). In the preceding cases we saw that Helen, Abby, and Molly all wrote answers at some point during their work on the task that did not address the prompt as written, but rather what they assumed the prompt was. For example, some errors stemmed from not realizing that the years in the table do not all have a gap of two or writing an expression with 100 as an exponent. All three teachers corrected their errors as they progressed through the task. Then,

the teachers anticipated students encountering difficulty with the aspects of the tasks in which the teachers themselves made mistakes. This suggests that an aspect of the teachers' KCS is based on and includes knowledge of their own errors with respect to the task, indicating they possess the mathematical understanding to complete the task as written.

One of the most striking differences among how teachers reviewed the Xbox task surfaced when considering the coherence between the teachers' doing of the task and the ways students would think about and engage in the task. Molly focused on the way students would need to think about the mathematical concepts inherent in the task and only completed aspects of the tasks as a way to articulate student thinking. Unlike Molly, Abby and Helen completed the tasks for themselves before talking about students. When Abby and Helen discussed students, they tended to focus on describing what students may do and struggle with, but they did not articulate *why* or what thinking would create the struggle. Frankie, in contrast, did not complete the task as designed and discussed things not related to student mathematical thinking. Of the four teachers, Molly showed the greatest integration between the mathematics of the task and the students' thinking related to the task. That is, this task review assessment elicited evidence of Molly's SCK/KCS for exponential functions that we were not able to elicit from Helen or Abby.

Conclusion

The MKT interview was useful for gathering insight into teachers' MKT related to KCS, KCT, and KCC. The insight comes from doing an interview that allowed for teachers to articulate their own thinking in regards to a non-traditional task rather than selecting predetermined answers. We hypothesize that the interactions within an MKT interview provide a more genuine representation of MKT and call for further investigation.

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